

2.5 Reason Using Properties from Algebra

Let a , b , and c be real numbers.

Properties of Equality

Addition Property--If $a = b$ and $c = d$, then $a + c = b + d$.

Subtraction Property--If $a = b$ and $c = d$, then $a - c = b - d$.

Multiplication Property--If $a = b$, then $ca = c \cdot b$.

Division Property-- If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$

Distributive Property-- $a(b + c) = ab + ac$

Substitution Property--If $a = b$, then either a or b may be substituted for the other in any equation.

** (also combining like terms)*

Reflexive Property-- $a = a$

Symmetric Property--If $a = b$, then $b = a$.

Transitive Property--If $a = b$ and $b = c$, then $a = c$.

Reflexive, symmetric, and transitive also work with **congruence**.

Example 1	Statements	Reasons
1.	$8(x - 5) = 32$	1. Given
2.	$8x - 40 = 32$	2. Distributive
3.	$8x - 40 + 40 = 32 + 40$	3. Addition
4.	$8x = 72$	4. Substitution
5.	$\frac{8x}{8} = \frac{72}{8}$	5. Division
6.	$x = 9$	6. Substitution

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4.	$x = 9$	4. Division

Example 2

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|------------------|----------|
| 1. $5x - 3 = 12$ | 1. Given |
| 2. $5x = 15$ | 2. Add. |
| 3. $x = 3$ | 3. Div |

Example 3

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|-----------------------|-----------|
| 1. $8x + 3x - 9 = 24$ | 1. Given |
| 2. $11x - 9 = 24$ | 2. Subst. |
| 3. $11x = 33$ | 3. Add. |
| 4. $x = 3$ | 4. Div. |

Example 4

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|--------------------------------|-----------|
| 1. $2(5 - 3x) - 4(x + 7) = 92$ | 1. Given |
| 2. $10 - 6x - 4x - 28 = 92$ | 2. Distr |
| 3. $-10x - 18 = 92$ | 3. Subst. |
| 4. $-10x = 110$ | 4. Add. |
| 5. $x = -11$ | 5. Div |
| 6. $-11 = x$ | 6. Sym. |

Justify each statement.

- Reflexive $m\angle 1 = m\angle 1$
- Addition If $m\angle 1 = m\angle 2$, then $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$
- Mult. If $AB = CD$, then $2 \cdot AB = 2 \cdot CD$.
- Symmetric If $RS = XY$, then $XY = RS$
- Transitive If $m\angle A = m\angle B$, and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.
- Division If $2 \cdot m\angle 1 = 90$, then $m\angle 1 = 45$
- Subst. If $m\angle 9 + m\angle 10 = 150^\circ$, and the $m\angle 10 = 48^\circ$, then $m\angle 9 + 48 = 150$.
- Subtr. If $m\angle 9 + 48 = 150$, then $m\angle 9 = 102$.

State the property that justifies each statement.

4. If $2x = 5$, then $x = \frac{5}{2}$. **Div.**
 5. If $\frac{x}{2} = 7$, then $x = 14$. **Mult.**
 6. If $x = 5$ and $b = 5$, then $x = b$. **Subst.**
 7. If $XY - AB = WZ - AB$, then $XY = WZ$. **Add.**
 8. Solve $\frac{x}{2} + 4x - 7 = 11$. List the property that justifies each step.
 9. Complete the following proof.

Given: $5 - \frac{2}{3}x = 1$

Prove: $x = 6$

Proof:

Statements	Reasons
a. $5 - \frac{2}{3}x = 1$	a. Given
b. $3(5 - \frac{2}{3}x) = 3(1)$	b. Mult.
c. $15 - 2x = 3$	c. Distr.
d. $-2x = -12$	d. Subtraction Prop.
e. $x = 6$	e. Div.

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p108-109
3-7, 9, 21-25,

WRITING REASONS Copy the logical argument. Write a reason for each step.

3. $3x - 12 = 7x + 8$ **Given**
 $-4x - 12 = 8$ **?**
 $-4x = 20$ **?**
 $x = -5$ **?**
4. $5(x - 1) = 4x + 13$ **Given**
 $5x - 5 = 4x + 13$ **?**
 $x - 5 = 13$ **?**
 $x = 18$ **?**

5. ★ **MULTIPLE CHOICE** Name the property of equality the statement illustrates: If $XY = AB$ and $AB = GH$, then $XY = GH$.
 (A) Substitution (B) Reflexive (C) Symmetric (D) Transitive

WRITING REASONS Solve the equation. Write a reason for each step.

6. $5x - 10 = -40$
 7. $4x + 9 = 16 - 3x$
 9. $3(2x + 11) = 9$

COMPLETING STATEMENTS In Exercises 21–25, use the property to copy and complete the statement.

21. Substitution Property of Equality: If $AB = 20$, then $AB + CD = \underline{\quad ? \quad}$.
 22. Symmetric Property of Equality: If $m\angle 1 = m\angle 2$, then $\underline{\quad ? \quad}$.
 23. Addition Property of Equality: If $AB = CD$, then $\underline{\quad ? \quad} + EF = \underline{\quad ? \quad} + EF$.
 24. Distributive Property: If $5(x + 8) = 2$, then $\underline{\quad ? \quad}x + \underline{\quad ? \quad} = 2$.
 25. Transitive Property of Equality: If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $\underline{\quad ? \quad}$.